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COUPLING CONSTANTS OF SPIN-TWO MESONS WITH TWO PSEUDOSCALAR MESONS

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ABSTRACT

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Recently discovered spin-two mesons f, A_2, f'(1500), K**(1405) are considered as resonances in various systems containing two pseudoscalar mesons with appropriate quantum numbers. Making the single channel approximation, exact N/D calculations are done by solving the integral equations with the help of matrix inversion method. The input forces due to exchanges of vector and spin-two mesons are taken into account. The input coupling constants given by experiments or SU(3) symmetry are used. By adjusting the straight cutoffs on the integrals, resonances are produced in each of these systems at the experimentally observed positions, and the output coupling constants are obtained. As expected in such calculations, the output coupling constants come out to be larger than the input ones. However, an interesting feature is found that the hierarchy of the coupling constants given by experiments or SU(3) symmetry is maintained.

I. INTRODUCTION

Recent experiments have indicated existence of several meson resonance having spin two and positive parity.[1,2] Among these, spin-parity of f(1250) and $A_2(1320)$ have been rather well established, whereas for K^{**} (1405) and f' (1500) these quantum numbers have been shown to be very likely. These mesons have been found to decay into several modes consisting of pseudoscalar (PS) and vector (V) mesons. There is also considerable support to the assignment of these particles to a reducible SU(3) nonet.[3,4] Existence of spin-two (T) mesons is also supported by rough bootstrap calculations.[5,6,7] Thus it seems to be of interest to perform detailed dynamical calculations of these systems.

At the very outset it is clear that even if we restrict ourselves to the open channels only, this is going to be a complicated multichannel calculation. Also some of the closed channels should be taken into account since they may help in narrowing the predicted widths of the resonances. Besides, the resonances are at fairly high energies, so the inelastic effects may be quite important. However, for the sake of simplicity we have done single channel calculations for some of the two PS meson systems in relative D wave states and having the lowest thresholds for the appropriate quantum numbers. Such a calculation can of course be improved in the future by including more PS meson channels and particularly channels containing two vector mesons or one vector and one PS meson. Within the single channel approximation , however, we have solved the problem by using the exact N/D method, so that the solutions are independent of the subtraction point. As the exchanged particles are not Reggeized, the usual divergence difficulties occur which make the integral equation for N non-Fredholm type. These are met with by introducing straight cutoffs which are adjusted to yield resonances at the given experimental masses. The equations are solved by the matrix inversion method described in detail, for example, by Fulco, Shaw and Wong.[8]

For the input forces, we use the single particle exchange diagrams involving both V and T mesons which are under study.[9] Input masses and coupling constants are taken from experiments whenever available. For coupling constants not available in this manner the values given by SU(3) symmetry are used. We have not made any attempt to bootstrap the system. This seems to be rather a remote possibility in view of the past experience with mesonic systems, where the output widths or coupling constants come out to be much larger than any realistic input ones. We are rather interested in predicting output values of coupling constants from reasonable values of the input constants.

For the sake of completeness, even within the framework of single particle exchange potentials, one should consider forces due to exchange of other possible mesonic resonances. In fact it is likely that many resonances with $J^P = 0^+$, 1^+ or 2^- may exist. Out of these, 1^+ and 2^- mesons happily cannot couple to two PS meson systems. Furthermore, for reasonable values of coupling constants, one can verify that forces due to exchange of 0^+ mesons are considerably weak. Hence they can be neglected without affecting the results of the present calculation significantly.[10]

With the above remarks we now turn to the details of the calculation. In Sec. II, in order to avoid repetitions for each case, we give general expressions for Born terms due to exchange of 1^- and 2^+ mesons in t and u channels for the s channel elastic scattering of two PS mesons of masses m_1 and m_2 (only such processes are considered in the present work). In Sec. III we consider in turn various PS meson systems in different I - spin and relative D wave states. The spin two mesons have been considered as resonances in two PS meson systems having the lowest thresholds for the appropriate quantum numbers and which appear to have strongest couplings to the resonances either on experimental grounds or from the point of view of SU(3) symmetry. Thus we have regarded K^{**} as a resonance in $I = \frac{1}{2}$ $K - \pi$ system, f in $I = 0$ $\pi - \pi$ system and f' in $I = 0$ $K - \bar{K}$ system. For A_2 we have considered both $I = 1$ $\eta - \pi$ and $K - \bar{K}$ systems separately. The relevant crossing matrix elements and coupling constants are given for all the cases. In Sec. IV the solution of N/D equations and results are discussed. It is found that by employing cutoff energies of about 25 to 45 m_π, it is possible to produce all the experimentally observed resonances in these systems. As expected, the output widths or coupling constants are much larger than the input ones. However the hierarchy of these seems to be maintained. Some remarks are also made regarding the other PS meson channels not considered explicitly and couplings to channels containing vector mesons.

II. BORN TERMS: GENERAL CASE

Consider the elastic scattering of two pseudoscalar mesons of masses m_1 and m_2.(11) As usual we define the $\ell = 2$ partial wave scattering amplitude free from kinematic singularities and having the correct threshold behavior by

$$A(s) = \frac{T(s)}{8\pi q^4} \tag{1}$$

where $T(s)$ is the conventional T-matrix element and the normalization is such that the unitarity relation in the elastic scattering region becomes

$$\mathrm{Im}\, A^{-1}(s) = -\frac{q^5}{\sqrt{s}}\, \theta\,(s - s_{th}) \tag{2}$$

We assume that (2) holds good up to the cutoff value of s.

The momentum q and the invariants t and u are given by

$$q^2 = \left[s - (m_1 + m_2)^2 \right] \left[s - (m_1 - m_2)^2 \right] \Big/ 4s$$

$$t = -2q^2 (1 - \cos\theta) \tag{3}$$

$$u = 2m_1^{\,2} + 2m_2^{\,2} - s + 2q^2 (1 - \cos\theta)$$

Now we give the Born terms due to exchange of 1^- and 2^+ mesons.(12,13) Our notation is as follows: In the Born terms $B^{T}_{Mm_1 m_2}$, T indicates the

2^+ resonance in the s - channel, M the exchanged particle (or mass) and m_1 and m_2 the external PS mesons.

(i) Vector meson exchange in t - channel:

$$B^T_{M_t m_1 m_2}(s) = \frac{C^T_{M_t m_1 m_2}}{4\pi} \cdot \frac{2(a + d)\, Q_2(d)}{q^4} \tag{4}$$

Here we have

$$a = \left[s - m_1{}^2 - m_2{}^2 - q^2 \right] \Big/ q^2 \tag{5}$$

$$d = 1 + M_t{}^2/2q^2 \tag{6}$$

$C^T_{M_t m_1 m_2}$ denotes the product of the relevant coupling constants and the crossing matrix element. This will be given for each case in the following section.

(ii) Vector meson exchange in u - channel:

$$B^T_{M_u m_1 m_2}(s) = \frac{C^T_{M_u m_1 m_2}}{4\pi} \cdot \frac{2(h + e) Q_2(e)}{q^4} \tag{7}$$

where

$$h = 1 + \frac{s - (m_1{}^2 - m_2{}^2)^2 / M_u{}^2}{2q^2} \tag{8}$$

$$e = -1 + \frac{M_u^{\,2} - 2m_1^{\,2} - 2m_2^{\,2} + s}{2q^2} \qquad (9)$$

(iii) Spin-two meson exchange in t - channel:

$$B^T_{M_t m_1 m_2}(s) = \frac{C^T_{M_t m_1 m_2}}{4\pi} \frac{1}{16q^2} \Big[\frac{12}{35} Q_4\,(d) + \frac{11}{21} Q_2\,(d) + \frac{2}{15} Q_0\,(d) +$$
$$(3a + b) \left\{ \frac{3}{5} Q_3\,(d) + \frac{2}{5} Q_1\,(d) \right\} + \left(\frac{3a^2 - c}{2} \right) Q_2\,(d) \Big] \qquad (10)$$

where

$$b = 1 + \frac{m_1^{\,2} + m_2^{\,2}}{q^2} \qquad (11)$$

$$c = \left(1 + \frac{2m_1^{\,2}}{q^2}\right)\left(1 + \frac{2m_2^{\,2}}{q^2}\right) \qquad (12)$$

(iv) Spin-two meson exchange in u - channel:

$$B^T_{M_u m_1 m_2}(s) = \frac{C^T_{M_u m_1 m_2}}{4\pi} \frac{1}{16q^2} \Big[\frac{12}{35} Q_4\,(e) + \frac{11}{21} Q_2\,(e) + \frac{2}{15} Q_0\,(e) +$$
$$(4h - 2) \left\{ \frac{3}{5} Q_3\,(e) + \frac{2}{5} Q_1\,(e) \right\} + (h^2 + 2h - 2)\, Q_2\,(e) \Big] \qquad (13)$$

In the above expressions Q_ℓ's are the usual Legendre polynomials of the second kind. Having considered the Born terms for the general case, we now proceed in the next section to discuss the particular cases.

III. CROSSING MATRIX ELEMENTS AND COUPLING CONSTANTS

Before discussing the actual cases, it will be useful to note certain relations between various coupling constants[14] that will be needed in the following.

As is well known, the V - PS - PS meson coupling constants in the limit of exact SU(3) symmetry are related by

$$g_{\rho\pi\pi}^2 : g_{\rho K\bar{K}}^2 : g_{\omega_8 K\bar{K}}^2 : g_{K^*K\eta}^2 : g_{K^*K\pi}^2 = 4 : 2 : 6 : 3 : 3 \qquad (14)$$

Input values for $g_{\rho\pi\pi}^2$ and $g_{K^*K\pi}^2$ are taken from experimental widths of these resonances. For other coupling constants (14) is used since symmetry breaking effects in (14) do not seem to be important. Since only the octet component of the ω - ϕ system can be coupled to two PS mesons, it has been considered to be the exchanged particle in the following with a mass given by the Gell-Mann-Okubo mass formula.

Among others, Glashow and Socolow[4] have pointed out that the spin-two mesons can be assigned to a reducible nonet of SU(3) in which K** and A_2 are members of octet and f and f' are mixtures of their singlet and octet components. Since only symmetric coupling is allowed, one obtains for the pure octet states,

$$\gamma_{A_2\pi\eta}^2 : \gamma_{A_2K\bar{K}}^2 : \gamma_{K^{**}K\pi}^2 : \gamma_{K^{**}K\eta}^2 = 4 : 6 : 9 : 1 \qquad (15)$$

For the mixed f and f' states, following GS we have

$$\gamma_{f\pi\pi}^2 : \gamma_{fK\bar{K}}^2 : \gamma_{f\eta\eta}^2 : \gamma_{f'\pi\pi}^2 : \gamma_{f'K\bar{K}}^2 : \gamma_{f'\eta\eta}^2 = 35.7 : 15.2 : 2.1 : 0.3 : 20.8 : 9.9 \tag{16}$$

Input values for $\gamma_{f\pi\pi}^2$, $\gamma_{K^{**}K\pi}^2$, $\gamma_{A_2K\bar{K}}^2$, $\gamma_{A_2\pi\eta}^2$ are taken from the experimental widths available at present. For other coupling constants (15) and (16) are used. In this case, however, symmetry breaking effects may be important but fortunately as we shall see in the following, this does not affect our major conclusions. Now we consider resonances in different two PS meson systems.

(a) K^{**} (1405) ($I = \frac{1}{2}$ $K - \pi$ channel)

In this case ρ, f and f' can be exchanged in t channel while K^* and K^{**} occur in the u channel. The 'C' factors are given by the following:

$$\begin{aligned}
C^{K^{**}}_{\rho K\pi} &= \sqrt{2}\, g_{\rho K\bar{K}}\, g_{\rho\pi\pi} \\
C^{K^{**}}_{K^*K\pi} &= \frac{1}{3}\, g_{K^*K\pi}^2 \\
C^{K^{**}}_{fK\pi} &= \sqrt{\frac{1}{3}}\, \gamma_{f\pi\pi}\, \gamma_{fK\bar{K}} \\
C^{K^{**}}_{K^{**}K\pi} &= -\frac{1}{3}\, \gamma_{K^{**}K\pi}^2
\end{aligned} \tag{17}$$

Effects of f' exchange are neglected as it seems to be extremely weakly coupled to π-π system, besides being of higher mass. From (16) it can be seen that its contribution will be less than 10% of that due to the f exchange.

(b) A_2 (1320) (η - π channel)

Here we have to consider only exchange of f in t channel and A_2 in u channel. For the C's we have

$$C^{A_2}_{f\eta\pi} = \frac{2}{\sqrt{3}} \gamma_{f\pi\pi} \gamma_{f\eta\eta}$$

$$C^{A_2}_{A_2\eta\pi} = \gamma_{A_2\eta\pi}{}^2 \tag{18}$$

Again contribution of f' is neglected.

(c) A_2 (1320) (I = 1 K - $\bar{K}$ channel)

Vector mesons ρ and ω_8 as well as f, f' and A_2 can be exchanged in t channel. Again C's are given by

$$C^{A_2}_{\rho K\bar{K}} = - \tfrac{1}{2} g_{\rho K\bar{K}}{}^2$$

$$C^{A_2}_{\omega_8 K\bar{K}} = + \tfrac{1}{2} g_{\omega_8}{}^2{}_{K\bar{K}}$$

$$C^{A_2}_{fK\bar{K}} = \tfrac{1}{2} \gamma_{fK\bar{K}}{}^2 \tag{19}$$

$$C^{A_2}_{f'K\bar{K}} = \frac{1}{2} \gamma_{f'K\bar{K}}^2$$

$$C^{A_2}_{A_2K\bar{K}} = - \frac{1}{2} \gamma_{A_2K\bar{K}}^2$$

(d) f (1250) (I = 0 $\pi - \pi$ channel)

In this case ρ and f exchanges give

$$C^{f}_{\rho\pi\pi} = 2 g_{\rho\pi\pi}^2$$

$$C^{f}_{f\pi\pi} = \frac{2}{3} \gamma_{f\pi\pi}^2 \tag{20}$$

(e) f' (1500) (I = 0 K - $\bar{K}$ channel)

The C's due to exchanges of ρ , ω_8, f, f' and A_2 are given by

$$C^{f'}_{\rho K\bar{K}} = \frac{3}{2} g_{\rho K\bar{K}}^2$$

$$C^{f'}_{\omega_8 K\bar{K}} = \frac{1}{2} g_{\omega_8 K\bar{K}}^2$$

$$C^{f'}_{fK\bar{K}} = \frac{1}{2} \gamma_{fK\bar{K}}^2 \tag{21}$$

$$C^{f'}_{f'K\bar{K}} = \frac{1}{2} \gamma_{f'K\bar{K}}^2$$

$$C^{f'}_{A_2K\bar{K}} = \frac{3}{2} \gamma_{A_2K\bar{K}}^2$$

Using results of the present and the last section, Born terms for all the processes under consideration are obtained. In all the cases the relative signs of the coupling constants given by SU(3) symmetry are used. The resulting N/D equations are discussed in the following section.

IV. SOLUTION OF N/D EQUATIONS AND RESULTS

Writing $A(s) = N(s)/D(s)$ we have the usual equations for $N(s)$ and $D(s)$:

$$N(s) = B(s) + \frac{1}{\pi} \int_{s_{th}}^{s_c} \frac{ds' \, q'^5 \left[B(s') - \left(\frac{s - s_0}{s' - s_0} \right) B(s) \right] N(s')}{\sqrt{s'}\,(s' - s)} \tag{22}$$

$$D(s) = 1 - \frac{s - s_0}{\pi} \int_{s_{th}}^{s_c} \frac{ds' \, q'^5 N(s')}{\sqrt{s'}\,(s' - s_0)\,(s' - s)} \tag{23}$$

where $B(s)$ is the Born term and s_0, s_{th} and s_c are respectively the subtraction point, threshold and cut off. Integral equation (22) is of the Fredholm type and can be solved by employing the matrix inversion method.[8] Using the Born terms given above we have solved the set of equations (22) and (23) numerically. For each case the cutoff was varied till the curve for output cross section versus s yielded a peak at the experimental value for the resonance. The half widths at half the maximum on each side of the peak were then evaluated. Since for a wide resonance the position of peak in the cross - section may be considerably different from the zero of $D(s)$, cutoff was also varied till $D(s')$ developed zero at the position of resonance and the out-put coupling constant was obtained by taking the derivative of $D(s)$ at the position of its zero. It was found that the values of cutoffs required in the two cases were slightly different but the output values of the coupling constants or widths were not appreciably different. Numerical accuracy of the method was tested by verifying the stability of the results to the variation of the mesh size used to solve (22) and the subtraction point. Some of the results are given in Table I.

From the results we can see that strongly attractive forces do exist in these systems and by employing cutoff energies of about 25 to 45 m_π, it is possible to produce experimentally observed spin-two resonances. However the predicted widths are much larger than the experimental widths. In fact in such a crude one channel model with elastic unitary, one can hardly expect to get better results than the corresponding calculations for vector mesons, where large out-put widths are found. On the other hand it is interesting to note that the hierarchy of widths or coupling constants given by experiments or SU(3) seems to be maintained in our model.

For example, for the pure octet states, (15) gives

$$\gamma_{A_2\pi\eta}{}^2 < \gamma_{A_2K\bar{K}}{}^2 < \gamma_{K^{**}K\pi}{}^2$$

Experimental situation regarding the partial decay widths of $A_2 \to K\bar{K}$ and $\eta\pi$ is not clear at the moment. However, these are certainly less than $\Gamma_{K^{**}\to K\pi}$. If we use for $\gamma^{\text{in}\,2}_{A_2\eta\pi}$ the smaller value given by experiments at present (0.03) we obtain $\gamma^{\text{out}}_{A_2\eta\pi} < \gamma_{A_2K\bar{K}}$ whereas for the larger input value given by SU(3) (0.06) the two become comparable, but still less than $\gamma^{\text{out}}_{K^{**}K\pi}$ (of course these results may be modified in a two channel $\eta\pi$ - $K\bar{K}$ calculation). The input width of $K^{**} \to K\pi$ was at first assumed to be 75 Mev from GS. We tried also a smaller width (= 42 Mev given by GS from SU (3)). This increased $\gamma^{out}_{K^{**}K\pi}$ by about 5%, thus improving the validity of the above inequality. According to GS, $\gamma_{f\pi\pi}{}^2$ is the largest of all the T-PS-PS meson coupling constants. This holds good in our model too. Regarding couplings of f' the experimental situation is again not clear but our present model predicts that

$$\gamma_{K^{**}K\pi}^{\;2} < \gamma_{f'K\bar{K}}^{\;2} < \gamma_{f\pi\pi}^{\;2}$$

a relation which holds good in the mixing model of GS! (Increase of input value of $\gamma_{f'K\bar{K}}$ affects the result given in Table I only slightly.) It should be noted that only the results for $\gamma_{A_2\eta\pi}{}^2$ and $\gamma_{A_2K\bar{K}}{}^2$ are somewhat sensitive to the input values of the T-PS-PS coupling constants, most of which are rather poorly known at present. In the first case no known vector mesons (1^-) can be exchanged and in the second case their contribution is small due to the mutual cancellation. Incidentally, this fact is responsible for giving the output values of these coupling constants smaller than the rest. For the other cases vector meson exchanges give dominant contributions to the value of Born terms in low s region and as a consequence give larger output coupling constants. It must be stressed, however, that vanishing of the D function in the low energy region for such low cutoffs is possible essentially due to the large contribution of the spin-two meson exchange Born terms at high energies.

Now for some of the other channels not considered above we wish to make the following remarks. It can be shown that in I = 3/2 K - $\pi(2^+)$ channel both ρ and K* exchange give rise to strong repulsion and hence it will not be possible to produce K**. Thus our model strongly supports the assignment I = 1/2 to K**.

Assuming isospin of K** to be 1/2, we note that in K - η channel, K* exchange again gives rise to a repulsive force. Therefore a one channel calculation will not give any resonance, but there is a lower channel (K - π) and a two-channel calculation will most likely give rise to a resonance. Furthermore, since the force is also repulsive in the off-diagonal term, a simple consideration of the two channel N matrix suggests that $\gamma^{out}_{K^{**}K\eta}$ will quite likely turn out to be considerably smaller than $\gamma^{out}_{K^{**}K\pi}$. From (15) one can see than this is required by SU(3) symmetry.

Apart from the last remark on K**, it is not clear how the above results will be modified by a multichannel calculation. A more realistic calculation should take into account not only more two PS meson channels, but also two V mesons and one V and one PS meson. Inclusion of the latter may bring noticable difference between couplings of f and f'. This is because of the fact that the charge conjugation invariance allows only the antisymmetric coupling between the octet V and PS mesons in the 2^+ state. Hence these cannot be coupled to the singlet member of the T - meson multiplet.

In the present calculation we adopted a very simple one channel model for spin two mesons. In spite of this we have been able to produce experimentally observed resonances and obtain some interesting results about the coupling constants. That the output coupling constants come out to be much larger than input ones is a common disease of such calculations which we have not been able to correct. However, if some common mechanism such as long range repulsion due to the crossed channel Regge trajectories or contributions of baryon-antibaryon intermediate states is successful in reducing the predicted coupling constants in such calculations, then one can hope that again such regularities in hierarchy of coupling constants will be maintained. Finally we remark that the experimental situation regarding couplings of spin-two meson resonances is expected to become clearer in future. Then it will be certainly interesting to perform a multi-channel calculation for these systems.

References and Footnotes

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9. We note that the partial wave dispersion relations have no solution satisfying unitarity and correct threshold behavior if the angular momentum is larger than the largest spin of the exchanged particles. See, for example, A. P. Balachandran and F. V. Hippel, Ann. of Phys. 30, 446 (1964) or A. Martin, Nuovo Cimento 38, 1326 (1965).
10. This statement is made for the elementary particle exchange. Recently G. F. Chew (UCRL - 16245) has made an interesting proposal that such Reggeized particles (probably unphysical) give rise to long range repulsion which may help in reducing the predicted widths of mesonic resonances. We have not considered such a possibility.
11. Throughout we have used the symbols for the particle and its mass interchangeably. s is given in units of square of the pion mass.
12. In our conventions, the T-matrix element for spin two meson exchange in say t-channel is given by

$$\frac{\gamma_{M_t m_1 m_3} \gamma_{M_t m_2 m_4} q_t^{\,4} P_2 (\cos \theta_t)}{M_t^{\,2} - t}$$

apart from crossing matrix elements.

13. In obtaining u-channel Born terms, certain terms with poles at $u = 0$ occur. To avoid this unwanted singularity while projecting out the partial wave, we have substituted $u = M^2$ (mass of exchanged particle^2) in such terms but not in other terms where u occurs.

14. Our coupling constants are normalized in such a way that, for example, the widths of $\rho \to \pi + \pi$ and $f \to \pi + \pi$ are given by

$$\Gamma_\rho = \frac{8}{3} \frac{g_{\rho\pi\pi}^{\,2}}{4\pi} \frac{q_R^{\,3}}{\rho^2}, \quad \Gamma_f = \frac{1}{10} \frac{\gamma_{f\pi\pi}^{\,2}}{4\pi} \frac{q_R^{\,5}}{f^2}$$

q_R being the c. m. momentum of the particles at the resonance.

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TABLE I - Results .

Values of mass^2 and cutoff are given in units of $m_\pi^{\ 2}$ and width Γ is in Mev. GR_{in} and GR_{out} respectively represent the squares of the relevant input and output T - PS - PS coupling constants.

Resonance	Mass^2	cutoff	Γ_{tot}	GR_{in}	GR_{out}
$f_0(\pi\pi)$	82.4	565.	567.	0.36	2.0
$A_2(\eta\pi)$	92.	1143.	107.	0.03	0.75
$A_2(K\bar{K})$	92.	1135.	42.	0.1	0.89
$K^{**}(K\pi)$	103.1	2090.	231.	0.32	1.1
$K^{**}(K\pi)$	103.1	1230.	266.	0.18	1.2
$f'(K\bar{K})$	118.2	885.	186.	0.21	1.4